MARCH 2019 NUMBER 2 VOLUME 50

Journal for Research in Mathematics Education

NCTM® NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

VOLUME 50 | NUMBER 2 | MARCH 2019

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Journal for Research in Mathematics Education
2019, Vol. 50, No. 2, 114–120

Editorial

Posing Significant Research Questions

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In 2002, the National Research Council (NRC) released *Scientific Research in Education*, a report that proposed six principles to serve as guidelines for all scientific inquiry in education. The first of these principles was to "pose significant questions that can be investigated empirically" (p. 3). The report argued that the significance of a question could be established on a foundation of existing theoretical, methodological, and empirical work. However, it is not always clear what counts as a significant question in educational research or where such questions come from. Moreover, our analysis of the reviews for manuscripts submitted to *JRME*[1] suggests that some practical, specific guidance could help researchers develop a significant question or make the case for the significance of a research question when preparing reports of research for publication.

Building on the *JRME* archive of nearly 50 years of research articles, this issue marks the beginning of a series of editorials aimed at discussing how to conduct and report high-quality research in mathematics education. In this first editorial in the series, we discuss what counts as a significant research question in mathematics education research, where significant research questions come from, and how researchers can develop their manuscripts to make the case for the significance of their research questions. Although we are beginning a new series of editorials, we will continue to draw on the ideas from our editorials over the past 2 years (e.g., Cai et al., 2018; Cai et al., 2017). In particular, we consider what significant research questions might look like in the aspirational future world of research that we have described in those editorials—a world in which mathematics education research is carried out by widespread, stable partnerships of teachers and researchers and in which research both takes root in and shapes the everyday practices of mathematics teaching and learning.

Significant Research Questions

It is difficult, if not impossible, to judge the significance of a research question just by reading the question. Certainly, significant research in mathematics education should advance the field's knowledge and understanding of the teaching and learning of mathematics (Heid, 2010; Simon, 2004). We believe this implies that the characteristics that make a research question significant are dependent on

[1] We analyzed the reviews for every manuscript that underwent full review and received a decision in 2017. For those manuscripts that were ultimately rejected, not a single reviewer stated that the research questions were particularly relevant or insightful. In contrast, for those manuscripts that ultimately received a revise and resubmit decision or were accepted (pending revisions), only one reviewer raised the concern that the research questions would not make a contribution to the field.

Journal for Research in Mathematics Education
2019, Vol. 50, No. 2, 121–155

Linguistic Conventions of Mathematical Proof Writing at the Undergraduate Level: Mathematicians' and Students' Perspectives

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This study examined the genre of undergraduate mathematical proof writing by asking mathematicians and undergraduate students to read 7 partial proofs and identify and discuss uses of mathematical language that were out of the ordinary with respect to what they considered conventional mathematical proof writing. Three main themes emerged: First, mathematicians believed that mathematical language should obey the conventions of academic language, whereas students were either unaware of these conventions or unaware that these conventions applied to proof writing. Second, students did not fully understand the nuances involved in how mathematicians introduce objects in proofs. Third, mathematicians focused on the context of the proof to decide how formal a proof should be, whereas students did not seem to be aware of the importance of this factor.

Keywords: Mathematical language; Mathematicians; Proof; Undergraduate students

Activities related to the notion of mathematical proof (e.g., justifying a given mathematical claim, presenting those justifications to others, and reading and critiquing others' justifications) constitute a crucial mathematical practice, one that mathematics educators would like students to engage in at different educational levels. However, mathematicians and mathematics educators have found undergraduate mathematics students to have difficulties when constructing (Weber, 2001), reading (Conradie & Frith, 2000), and validating (Selden & Selden, 2003) mathematical proofs. According to Moore (1994), one of the three "major sources of the students' difficulties" (p. 249) with proof is their unfamiliarity with the language and notation of mathematical proof writing. However, neither this language nor how students understand it has been examined in a systematic and empirical way in undergraduate mathematics education research.

We assume that students' understanding of this technical language is determined by (a) their exposure to the language (e.g., in textbook authors' writing and

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This research was conducted as part of a doctoral dissertation at Rutgers University. Portions of this research were presented at the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, East Lansing, Michigan, November 2015; the 2016 AMS/MAA Joint Mathematics Meetings, Seattle, Washington, January 2016; and the 19th Annual Conference on Research in Undergraduate Mathematics Education, Pittsburgh, February 2016.

Journal for Research in Mathematics Education
2019, Vol. 50, No. 2, 156–209

Surveying Middle-Grades Teachers' Reasoning About Fraction Arithmetic in Terms of Measured Quantities

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We report a novel survey that narrows the gap between information about teachers' knowledge of fraction arithmetic provided, on the one hand, by measures practical to administer at scale and, on the other, by close analysis of moment-to-moment cognition. In particular, the survey measured components that would support reasoning directly with measured quantities, not by executing computational algorithms, to solve problems. These components—each of which was grounded in past research—were attention to referent units, partitioning and iterating, appropriateness, and reversibility. A second part of the survey asked about teachers' professional preparation and history. We administered the survey to a national sample of in-service middle-grades mathematics teachers in the United States and received responses from 990 of those teachers. We analyzed responses to items in the first part of the survey using the log-linear diagnostic classification model to estimate each teacher's profile of strengths and weaknesses with respect to the four components of reasoning. We report on the diversity of profiles that we found and on relationships between those profiles and various aspects of teachers' professional preparation and history. Our results provide insight into teachers' knowledge resources for enacting standards-based instruction in fraction arithmetic and an example of new possibilities for mathematics education research afforded by recent advances in psychometric modeling.

Keywords: Assessment; Diagnostic classification models; Fractions; Measurement; Psychometrics; Teacher knowledge

Recent curriculum standards (e.g., National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) and recommendations for teacher

The following members of the Diagnosing Teachers' Multiplicative Reasoning project helped draft items for the fractions survey at the center of the present study: Bridget Druken, Laura Giberson, Joanne Lobato, Cynthia Lopez, Jessica McCreary, Caitlin O'Connor, Chandra Orrill, Amanda Paganin, Becky Stephens, and Lauren Susoeff. Jonathan Templin and Allan Cohen contributed to the development of the psychometric models we used. We thank the many teachers who participated in interviews and helped us understand how to refine our items and manuscript reviewers who provided thoughtful and helpful feedback.

This research was supported by the National Science Foundation (NSF) under Grant No. DRL-0903411. The opinions expressed are those of the authors and do not necessarily reflect the views of NSF.