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CONTENTS

ENERGY DISPERSED SOLUTIONS FOR THE $(4+1)$-DIMENSIONAL MAXWELL-KLEIN-GORDON EQUATION

By SUNG-JIN OH and DANIEL TATARU

Abstract. This article is devoted to the mass-less energy critical Maxwell-Klein-Gordon system in $4+1$ dimensions. In earlier work of the second author, joint with Krieger and Sterbenz, we have proved that this problem has global well-posedness and scattering in the Coulomb gauge for small initial data. This article is the second of a sequence of three papers of the authors, whose goal is to show that the same result holds for data with arbitrarily large energy. Our aim here is to show that large data solutions persist for as long as one has small energy dispersion; hence failure of global well-posedness must be accompanied with a non-trivial energy dispersion.

Contents.

1. Introduction. This article is concerned with the mass-less energy critical Maxwell-Klein-Gordon system (MKG) in the $4+1$ dimensional Minkowski space $\mathbb{R}^{1+4}$ equipped with the standard Lorentzian metric $\mathbf{m} = \mathrm{diag}(-1,1,1,1,1)$ in the standard rectilinear coordinates $(x^0,\dots,x^4)$. This system is generated by adding a scalar field component to the standard Maxwell Lagrangian,

$$\mathcal{S}_{\mathrm{M}}[A_\alpha] := \int_{\mathbb{R}^{1+4}} \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}\, dx\, dt;$$

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THE LANGLANDS-SHAHIDI L-FUNCTIONS FOR BRYLINSKI-DELIGNE EXTENSIONS

By FAN GAO

Abstract. We firstly discuss properties of the L-groups for Brylinski-Deligne extensions of split reductive groups constructed by M. Weissman. Secondly, the Gindikin-Karpelevich formula for an arbitrary Brylinski-Deligne extension is computed and expressed in terms of naturally defined elements of the group. Following this, we show that the Gindikin-Karpelevich formula can be interpreted as Langlands-Shahidi type L-functions associated with the adjoint action of the L-group for the Levi covering subgroup on certain Lie algebras. As a consequence, the constant term of Eisenstein series for Brylinski-Deligne extensions could be expressed in terms of global (partial) Langlands-Shahidi type L-functions. These L-functions are shown to possess meromorphic continuation to the whole complex plane. In the end, we determine the residual spectra of Brylinski-Deligne covers of some semisimple rank one groups.

Contents.

1. Introduction.

1.1. Covering groups and L-groups. Recently, Brylinski and Deligne have developed quite a general theory of covering groups of algebraic nature in their influential paper [BD01]. In particular, they classified multiplicative $\mathbb{K}_2$-torsors $\overline{\mathbb{G}}$ (equivalently in another language, central extensions of $\mathbb{G}$ by $\mathbb{K}_2$) over an algebraic group $\mathbb{G}$ in the Zariski site of Spec(F). Such a central extension is written as

$$\mathbb{K}_2 \hookrightarrow \overline{\mathbb{G}} \twoheadrightarrow \mathbb{G},$$

which has kernel the sheaf $\mathbb{K}_2$ defined by Quillen. In fact, they actually work over general schemes and not necessarily Spec(F), but for our purpose we take this more restrictive consideration in this paper.

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ON THE ANALYTICITY OF CR-DIFFEOMORPHISMS

By I. KOSSOVSKIY and B. LAMEL

Abstract. In any positive CR-dimension and CR-codimension we provide a construction of real-analytic embedded CR-structures, which are C^∞ CR-equivalent, but are inequivalent holomorphically.

Contents.

1. Introduction.

1.1. Overview. Let M be a smooth manifold. A *CR-structure* on M is a couple $(\mathcal{B}, J)$, where $\mathcal{B}$ is an even-dimensional subbundle of TM (called *the complex tangent bundle* and denoted by $T^{\mathbb{C}}M$), and $J_p\colon \mathcal{B}_p \to \mathcal{B}_p$ is a linear operator with $J_p^2 = -\mathrm{Id}$, smoothly depending on $p \in M$ and called the *complex structure operator*. A manifold M endowed with a CR-structure is called an *(abstract) CR-manifold*. If the CR-structure is *integrable* [38] (e.g., if it is real-analytic), then there exists a (local) generic embedding $\mathrm{i}\colon M \hookrightarrow \mathbb{C}^N$ such that the complex tangent bundle of M is given by the $T^{\mathbb{C}}M = TM \cap iTM$, and the complex structure J is induced by multiplication with i in $\mathbb{C}^N$. Natural morphisms of CR-structures are *CR-maps*, that is, smooth maps $F\colon M \to M$ such that $F^*\mathcal{B}' = \mathcal{B}$ and $dF \circ J = J' \circ dF$. For CR-manifolds embedded into $\mathbb{C}^N$ (*CR-submanifolds*)

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KÄHLER-RICCI FLOW WITH UNBOUNDED CURVATURE

By SHAOCHUANG HUANG and LUEN-FAI TAM

Abstract. Let $g(t)$ be a smooth complete solution to the Ricci flow on a noncompact manifold such that $g(0)$ is Kähler. We prove that if $|\mathrm{Rm}(g(t))|_{g(t)}$ is bounded by a/t for some $a > 0$, then $g(t)$ is Kähler for $t > 0$. We prove that there is a constant $a(n) > 0$ depending only on n such that the following is true: Suppose $g(t)$ is a smooth complete solution to the Kähler-Ricci flow on a noncompact n-dimensional complex manifold such that $g(0)$ has nonnegative holomorphic bisectional curvature and $|\mathrm{Rm}(g(t))|_{g(t)} \le a(n)/t$, then $g(t)$ has nonnegative holomorphic bisectional curvature for $t > 0$. These generalize the results by Wan-Xiong Shi. As applications, we prove that (i) any complete noncompact Kähler manifold with nonnegative complex sectional curvature and maximum volume growth is biholomorphic to $\mathbb{C}^n$; and (ii) there is $\epsilon(n) > 0$ depending only on n such that if (M^n, g_0) is a complete noncompact Kähler manifold of complex dimension n with nonnegative holomorphic bisectional curvature and maximum volume growth and if $(1+\epsilon(n))^{-1}h \le g_0 \le (1+\epsilon(n))h$ for some Riemannian metric h with bounded curvature, then M is biholomorphic to $\mathbb{C}^n$.

1. Introduction. In [25], Simon proved that there is a constant $\epsilon(n) > 0$ depending only on n such that if (M^n, g_0) is a complete n-dimensional Riemannian manifold and if there is another metric h with curvature bounded by k_0 and

$$(1+\epsilon(n))^{-1}h \le g_0 \le (1+\epsilon(n))h,$$

then the so-called h-flow has a smooth short time solution $g(t)$ such that

$$|\mathrm{Rm}(g(t))|_{g(t)} \le C/t. \tag{1.1}$$

Here h-flow is exactly the usual Ricci-DeTurck flow. We call it h-flow as in [25] for emphasizing the background metric h. For the precise definition of h-flow, see Section 5. The method by Schnürer-Schulze-Simon [21] can be carried over to construct Ricci flow using the above solution to the h-flow. On the other hand, in [2], Cabezas-Rivas and Wilking proved that if (M, g_0) is a complete noncompact Riemannian manifold with nonnegative complex sectional curvature, and if the volume of geodesic ball $B(x,1)$ of radius 1 with center at x is uniformly bounded below away from 0, then the Ricci flow has a smooth complete short time solution with nonnegative complex sectional curvature so that (1.1) holds. Recall that a Riemannian manifold is said to have nonnegative complex sectional curvature if $R(X,Y,\bar{Y},\bar{X}) \ge 0$ for any vectors X, Y in the complexified tangent bundle.

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ON IWAHORI-HECKE ALGEBRAS FOR p-ADIC LOOP GROUPS: DOUBLE COSET BASIS AND BRUHAT ORDER

By DINAKAR MUTHIAH

Abstract. We study the p-adic loop group Iwahori-Hecke algebra $\mathcal{H}(G^+,I)$ constructed by Braverman, Kazhdan, and Patnaik and give positive answers to two of their conjectures. First, we algebraically develop the "double coset basis" of $\mathcal{H}(G^+,I)$ given by indicator functions of double cosets. We prove a generalization of the Iwahori-Matsumoto formula, and as a consequence, we prove that the structure coefficients of the double coset basis are polynomials in the order of the residue field. The basis is naturally indexed by a semi-group $\mathcal{W}_{\mathcal{T}}$ on which Braverman, Kazhdan, and Patnaik define a preorder. Their preorder is a natural generalization of the Bruhat order on affine Weyl groups, and they conjecture that the preorder is a partial order. We define another order on $\mathcal{W}_{\mathcal{T}}$ which carries a length function and is manifestly a partial order. We prove the two definitions coincide, which implies a positive answer to their conjecture. Interestingly, the length function seems to naturally take values in $\mathbb{Z}\oplus\mathbb{Z}\varepsilon$ where ε is "infinitesimally" small.

1. Introduction. Let $\mathbf{G}$ be a Kac-Moody group equipped with a choice of positive Borel subgroup $\mathbf{B}$. Let F be a non-archimedean local field, $\mathcal{O}$ be its ring of integers, π be a choice of uniformizer, and k be the residue field of $\mathcal{O}$. Let $G=\mathbf{G}(F)$, let $K=\mathbf{G}(\mathcal{O})$, and let the Iwahori subgroup I be those elements of K that lie in $\mathbf{B}(k)$ modulo the uniformizer.

When $\mathbf{G}$ is finite-dimensional, the Iwahori-Hecke algebra $\mathcal{H}(G,I)$ is defined to be the set of complex valued functions on G that are I-biinvariant and supported on finitely many I double cosets. The multiplication in $\mathcal{H}(G,I)$ is given by convolution. The I double cosets of G are indexed by the affine Weyl group. The "double coset basis" of $\mathcal{H}(G,I)$ is given by indicator functions of I double cosets, and the structure coefficients of this basis are given by the Iwahori-Matsumoto presentation of the algebra. Alternatively, Bernstein gave another presentation of $\mathcal{H}(G,I)$ by making use of the principal series representation of G. In this presentation, $\mathcal{H}(G,I)$ is generated by the Hecke algebra for the finite group $\mathbf{G}(k)$ and the group algebra of the coweight lattice of G.

In the case when $\mathbf{G}$ is an untwisted affine Kac-Moody group, i.e., a "loop group", the definition of Iwahori-Hecke algebra due to Braverman, Kazhdan and Patnaik [5] is more subtle. An initial issue is that the Cartan decomposition no longer holds. To handle this, let G^+ be the subset of G where the Cartan decomposition *does* hold, and restrict attention to only those I-biinvariant functions whose support is contained in G^+. Then one can prove that G^+ is in fact a semi-group,

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HYBRID IWASAWA ALGEBRAS AND THE EQUIVARIANT IWASAWA MAIN CONJECTURE

By Henri Johnston and Andreas Nickel

Abstract. Let p be an odd prime. We give an unconditional proof of the equivariant Iwasawa main conjecture for totally real fields for an infinite class of one-dimensional non-abelian p-adic Lie extensions. Crucially, this result does not depend on the vanishing of the relevant Iwasawa μ-invariant.

1. Introduction. Let p be an odd prime. Let K be a totally real number field and let K_∞ be the cyclotomic $\mathbb{Z}_p$-extension of K. An admissible p-adic Lie extension $\mathcal{L}$ of K is a Galois extension $\mathcal{L}$ of K such that (i) $\mathcal{L}/K$ is unramified outside a finite set of primes S of K, (ii) $\mathcal{L}$ is totally real, (iii) $\mathcal{G} := \mathrm{Gal}(\mathcal{L}/K)$ is a compact p-adic Lie group, and (iv) $\mathcal{L}$ contains K_∞. Let $M_S^{\mathrm{ab}}(p)$ be the maximal abelian p-extension of $\mathcal{L}$ unramified outside the set of primes above S. Let $\Lambda(\mathcal{G}) := \mathbb{Z}_p[[\mathcal{G}]]$ denote the Iwasawa algebra of $\mathcal{G}$ over $\mathbb{Z}_p$ and let X_S denote the (left) $\Lambda(\mathcal{G})$-module $\mathrm{Gal}(M_S^{\mathrm{ab}}(p)/\mathcal{L})$. Roughly speaking, the equivariant Iwasawa main conjecture (EIMC) relates X_S to special values of Artin L-functions via p-adic L-functions. This relationship can be expressed as the existence of a certain element in an algebraic K-group; it is also conjectured that this element is unique.

There are at least three different versions of the EIMC. The first is due to Ritter and Weiss and deals with the case where $\mathcal{G}$ is a one-dimensional p-adic Lie group [RW04], and was proven under a certain "$\mu = 0$" hypothesis in a series of articles culminating in [RW11]. The second version follows the framework of Coates, Fukaya, Kato, Sujatha and Venjakob [CFK+05] and was proven by Kakde [Kak13], again assuming $\mu = 0$. This version is for $\mathcal{G}$ of arbitrary (finite) dimension and Kakde's proof uses a strategy of Burns and Kato to reduce to the one-dimensional case (see Burns [Bur15]). Finally, Greither and Popescu [GP15] have formulated and proven an EIMC via the Tate module of a certain Iwasawa-theoretic abstract 1-motive, but they restricted their formulation to one-dimensional abelian extensions and the formulation itself requires a $\mu = 0$ hypothesis. In [Nic13], the second named author generalized this formulation (again assuming $\mu = 0$) to the one-dimensional non-abelian case, and in the situation that all three formulations make sense (i.e., $\mathcal{G}$ is a one-dimensional p-adic Lie group and $\mu = 0$), he showed

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