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Theoretical Framing as Justifying

Jinfa Cai, Anne Morris, Charles Hohensee, Stephen Hwang, Victoria Robison, Michelle Cirillo, Steven L. Kramer, and James Hiebert University of Delaware

In our March editorial (Cai et al., 2019), we discussed the nature of significant research questions in mathematics education. We asserted that the choice of a suitable theoretical framework is critical to establishing the significance of a research question. In this editorial, we continue our series on high-quality research in mathematics education by elaborating on how a well-constructed theoretical framework strengthens a research study and the reporting of research for publication. In particular, we describe how the theoretical framework provides a connecting thread that ties together all of the parts of a research report into a coherent whole. Specifically, the theoretical framework should help (a) make the case for the purpose of a study and shape the literature review; (b) justify the study design and methods; and (c) focus and guide the reporting, interpretation, and discussion of results and their implications.

JRME reviewers frequently comment on theoretical frameworks in their evaluations of manuscripts. Our analysis of the reviews for every manuscript that underwent full review and received a decision in 2017 revealed that reviewers raised concerns related to the theoretical framework in nearly 90% of manuscripts that were ultimately rejected. Indeed, approximately 70% of the individual reviews for these manuscripts included concerns related to the theoretical framework. Even for those manuscripts that were ultimately accepted, nearly 30% of the individual reviews still raised such concerns. Common concerns expressed by reviewers included the following: that the manuscript lacks a sufficiently developed framework, that the framework is not appropriate, that the framework is overly broad or generic, that the framework is overly narrow or myopic, and that the framework is disconnected from the other parts of the study. Concerns like these often reflect serious issues with a manuscript that generally require significant revisions if these concerns are to be effectively addressed.

What Is a Theoretical Framework?

Much has been written about theoretical frameworks, and some researchers have explicitly called for increased attention to theoretical frameworks in mathematics education research (e.g., Leatham, in press; Lester, 2005; Silver & Herbst, 2004; Skott, Van Zoest, & Gellert, 2013; Spangler & Williams, in press). Despite these calls, the notion of a theoretical framework can remain somewhat mysterious and confusing for novice and experienced researchers alike. Moreover, novice researchers may mistakenly believe that a theoretical framework is merely a straightforward summary of related studies. We recognize that some researchers make an explicit distinction between theoretical frameworks and conceptual frameworks (e.g., Eisenhart, 1991; Imenda, 2014; Lester, 2005). However, these

Two Undergraduate Students' Reinvention of the Multiplication Principle

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The multiplication principle (MP) is a fundamental aspect of combinatorial enumeration, serving as an effective tool for solving counting problems and underlying many key combinatorial formulas. In this study, we used guided reinvention to investigate 2 undergraduate students' reasoning about the MP, and we sought to answer the following research questions: How do students come to understand and make sense of the MP? Specifically, while a pair of students reinvented a statement of the MP, how did they attend to and reason about key mathematical features of the MP? The students participated in a paired 8-session teaching experiment during which they progressed from a nascent to a sophisticated statement of the MP. Two key mathematical features emerged for the students through this process, including independence and distinct composite outcomes, and we discuss ways in which these ideas informed the students' reinvention of the statement. In addition, we present potential implications and directions for future research.

Keywords: Combinatorics; Counting problems; Multiplication principle; Reinvention; Teaching experiment

The multiplication principle (MP), called by some the "fundamental principle of counting" (e.g., Richmond & Richmond, 2009), is a foundational aspect of combinatorial enumeration. Multiplication occurs as an appropriate operation in counting problems so frequently that many textbooks articulate a guiding principle for when to use multiplication in combinatorial settings. In this way, the MP serves as a characterization of when to use multiplication in counting. Broadly, it is the idea that for independent stages in a counting process, multiplying the number of options at each stage yields the total number of possible outcomes of the entire process. This principle underlies many basic counting formulas, providing justification for why these counting formulas work as they do. Despite its importance, relatively little has been studied concerning the MP itself. To better understand student thinking about the MP, we engaged two undergraduate students in a guided reinvention of a statement of the MP during an eight-session teaching experiment (Steffe & Thompson, 2000). In this article, we describe their overall reinvention process, present their progression of statements, and discuss two key mathematical features of the MP that arose for the students. Specifically, we seek to answer the following research questions: How do students come to understand and make sense of the MP? Specifically, while a pair of students reinvented a statement of the MP, how did they attend to and reason about key mathematical features of the MP? By answering these two questions, we can gain insight into how students engage with and reason about a foundational combinatorial idea. Because counting is known to be difficult for students generally (e.g., Batanero, Navarro-Pelayo, & Godino, 1997; Eizenberg & Zaslavsky, 2004; Hadar & Hadass,

A Qualitative Metasynthesis of Teaching Mathematics for Social Justice in Action: Pitfalls and Promises of Practice

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Mathematics classrooms are increasingly becoming sites for investigating social (in)justice, but research on teaching mathematics for social justice remains limited to individual case studies. This article reports on a metasynthesis of 35 qualitative reports of social justice mathematics enactments in diverse classroom contexts. Critical race theory serves as a guiding framework for analyzing possibilities and limitations of these enactments to address racial inequities in mathematics education. Findings from this metasynthesis reveal that addressing race in social justice mathematics explorations provided opportunities for centering the voices of people of Color and critiquing liberal views that camouflage subtle forms of racism and involved substantial and authentic mathematical work. Promising practices and implications for future research are identified based on this synthesis.

Keywords: Critical race theory; Mathematics for social justice; Metasynthesis; Race; Racism; Teaching Practice

With the sociopolitical turn in mathematics education (Gutiérrez, 2013), mathematics classrooms have increasingly become sites for critically investigating and reflecting on social justice issues. Some argue that teachers face a moral and ethical imperative to transform mathematics classrooms into spaces for the development of critical social awareness (i.e., critical consciousness) and social transformation (Stinson, 2014). Others have called for greater attention to the lives and experiences of those students, predominantly students of Color, whom mainstream research and reform efforts in mathematics education continue to marginalize (Berry, Ellis, & Hughes, 2014). In response, educators and scholars often target classrooms with students of Color, generally with low socioeconomic status (SES), as sites for reforming mathematics teaching to include social justice goals (Brantlinger, 2013).

Proponents of using mathematics to explore social justice issues and to advance change (i.e., teaching mathematics for social justice [TMSJ]) aim to place the best interests of students of Color at the center of reform efforts; however, tensions arise when teachers attempt to translate theories of TMSJ into practice. For example, time spent meaningfully discussing social issues naturally takes some time away from direct focus on mathematics, which may raise concerns among students of Color (e.g., Brantlinger, 2013). Moreover, in classrooms with white,

¹ For the remainder of this article, I use *students*, *teachers*, and *communities of Color* to refer to people from traditionally marginalized racial or ethnic groups. I recognize that this term has limitations (e.g., essentializes racial groups, suggests white as race or color neutral), but I adopt this term for the sake of clarity and readability. I chose to capitalize *Color* but not *white* to challenge the ways that these standard grammar conventions reinforce systems of privilege and oppression.

Mathematical Persistence Among Four African American Male Graduate Students: A Critical Race Analysis of Their Experiences

Christopher C. Jett University of West Georgia

The stories of high-achieving African American mathematics students are gaining prominence in the research literature. In this multiple case study, I use a critical race theoretical frame to document and analyze the experiences of 4 mathematically persistent African American male students who earned undergraduate degrees in mathematics and subsequently enrolled in mathematics or mathematics education graduate programs. The findings reveal that these African American men drew from internal factors to influence their mathematical persistence and identified how racial microaggressions manifest themselves in postundergraduate contexts. Recommendations for practice, policy implications, and future research directions that emerged from this study are discussed to better understand African American men's mathematics experiences.

Keywords: African American male students; Critical race theory; Mathematics; Qualitative research methods; Racial microaggressions

African American¹ male students have been the focal point of scholarly analyses, academic publications, and educational meetings in this contentious racial climate. With respect to Black men's undergraduate experiences, studies focus on how they experience and grapple with racism (Harper, 2013; Smith, Allen, & Danley, 2007). Black men accounted for approximately 2.4% of the undergraduate degrees awarded in mathematics and statistics during 2013–2014, and these numbers are exacerbated at the graduate level (U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, 2014). Despite their racial experiences and underrepresentation in mathematics, Black men continue to persist in the field (Jett, 2011; McGee & Martin, 2011a). Within a critical race theory (CRT) framework, I provide an analysis of the experiences of four Black men who completed their undergraduate degrees at historically Black

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¹ The terms African American and Black are used interchangeably throughout this manuscript and refer to students who were born in and completed their schooling in the United States.