

Journal for Research in Mathematics Education

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Editorial

So What? Justifying Conclusions and Interpretations of Data

Jinfa Cai, Anne Morris, Charles Hohensee, Stephen Hwang, Victoria Robison,
Michelle Cirillo, Steven L. Kramer, and James Hiebert
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Although often asked tactfully, a frequent question posed to authors by *JRME* reviewers is “So what?” Through this simple and well-known question, reviewers are asking: What difference do your findings make? How do your results advance the field? “So what?” is the most basic of questions, often perceived by novice researchers as the most difficult question to answer. Indeed, addressing the “so what” question continues to challenge even experienced researchers. All researchers wrestle with articulating a convincing argument about the importance of their own work. When we try to shape this argument, it can be easy to fall into the trap of making claims about the implications of our findings that reach beyond the data.

We use this editorial to propose some ideas for presenting and interpreting results with an eye toward addressing the “so what” question. We do so by leveraging the alignment among research questions, theoretical framework, and methods in a well-designed research study. Our aim is to present some practical ideas that could help researchers evaluate their findings with this question in mind.

Aligning Interpretations With Earlier Parts of the Report

In previous editorials, we argued that justifying the significance of a study requires developing a coherent chain of reasoning connecting the theoretical framework (Cai et al., 2019c), the research questions (Cai et al., 2019b), and the research methods chosen to address the research questions (Cai et al., 2019a). In this editorial, we argue that the chain of reasoning is not complete until the results are interpreted and discussed. The results do not stand alone; they fit within the story developed up to that point in the report. Therefore, the importance of the findings—the answer to the “so what” question—depends on the story developed before the results are presented. The importance of the findings, and of the study itself, emerges from interpreting the findings in a way that explicitly connects the data to earlier links in the chain.

Connecting Interpretations With the Research Questions

A first suggestion for connecting interpretations with research questions is that authors carefully consider how their findings address the research questions. Although this might seem like an obvious step in interpreting the data, authors often do not give it sufficient attention, perhaps because answering research questions is deceptively complex. The answers to research questions in mathematics education are (almost) never “yes” or “no.” Because educational settings are filled with interactions among multiple, and often confounding, factors, research questions that anticipate a yes or no answer hide important complexities. Appropriate

Brief Report

Mathematics Majors' Diagram Usage When Writing Proofs in Calculus

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We report on a study in which we observed 73 mathematics majors completing 7 proof construction tasks in calculus. We use these data to explore the frequency and effectiveness with which mathematics majors use diagrams when constructing proofs. The key findings from this study are (a) nearly all participants introduced diagrams on multiple tasks, (b) few participants displayed either a strong propensity or a strong reluctance to use diagrams, and (c) little correlation existed between participants' propensity to use diagrams and their mathematical achievement (either on the proof construction tasks or in their advanced mathematics courses). At the end of the report, we discuss implications for pedagogy and future research.

Keywords: Calculus; Diagrams; Proof; Visual reasoning

Although proof is expected to play a central role in elementary and secondary classrooms (e.g., National Council of Teachers of Mathematics, 2000; Schoenfeld, 1994), proof assumes even greater importance in advanced mathematics courses. In these courses, a primary goal of instruction is to increase students' ability to write proofs about the course content, and assessments of students' understanding of this content are largely composed of proving tasks.

Numerous studies suggest that mathematics majors have difficulty writing proofs, even after completing courses in advanced mathematics (e.g., Iannone & Inglis, 2010; Ko & Knuth, 2009; Moore, 1994; Weber, 2001; Weber, Alcock, & Radu, 2005). In each study, students were asked to complete a set of proving tasks, and their frequency of success was under 50%. In many of these studies, students' performance was alarmingly poor. For instance, Ko and Knuth (2009) found that none of the 36 mathematics majors in their study could successfully complete any of the three assigned proving tasks. Thus, an important aim of undergraduate mathematics education is to improve mathematics students' ability to successfully construct proofs.

One recommendation that some mathematics educators have made is that the use of diagrams can potentially help students construct proofs (e.g., Alcock & Simpson, 2004; Gibson, 1998; Schoenfeld, 1985). This recommendation is based on small-scale studies in which undergraduate students productively used diagrams to successfully complete proof construction tasks. Alternatively, Raman (2003) observed that, unlike mathematicians, some calculus students are reluctant

An A Priori Measure of Visual Difficulty of 2-D Sketches Depicting 3-D Objects

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Aiming to enhance understanding of visual obstacles inherent in two-dimensional (2-D) sketches used in high school spatial geometry instruction, we propose a measure of visual difficulty based on two attributes of the sketches: potentially misleading geometrical information (PMI) and potentially helpful geometrical information (PHI). The difficulty of 12 normatively oriented cube-related sketches was theoretically ranked according to their ratios, #PHI/#PMI. The ranking was compared to the actual visual difficulty as measured by the percentage of correct or desired comprehension, individual spatial ability, and study-time allocation. This procedure was repeated for unnormatively oriented sketches, obtained by vertically flipping the original sketches. In both cases, the findings substantiate #PHI/#PMI as an a priori measure of visual difficulty. Practical, theoretical, and methodological implications are inspected and discussed.

Keywords: High school; Measuring spatial visual difficulty; Normative and unnormative orientation of sketches; Prototypes; Spatial geometry instruction; Study-time allocation; Visual obstacles

Mathematics curricula in many countries tend to emphasize the importance of studying spatial geometry as an essential part of mathematical literacy and as a means of developing spatial aptitude (Bakó, 2003; McGee, 1979; Organisation for Economic Co-operation and Development [OECD], 2004; Parzysz, 1988). Yet, there is overwhelming evidence that spatial geometry suffers from dereliction in schools and presents a serious challenge for both high school teachers and students (Bakó, 2003; Gutiérrez, 1996; Parzysz, 1988). High school students' poor performance on tasks requiring spatial abilities on Programme for International Student Assessment [PISA] exams (OECD, 2004) indicates that spatial geometry instruction is problematic in many OECD countries, including Israel, where this study was conducted.

Impediments to teaching spatial geometry have been documented and investigated in several countries. For example, Ferrarello, Mammana, and Pennisi (2014) found that, in Italy, teaching spatial geometry is often relegated to the end of the

The work reported in this article is based on doctoral dissertation research conducted by the first author under supervision of the second and the third author at the Technion – Israel Institute of Technology. A short version of this article was presented at the PME 38th conference (Widder, Berman, & Koichu, 2014). The authors would like to thank Nadia Bordo for her help with the statistical analysis.

Who Gets In? Examining Inequality in Eighth-Grade Algebra

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Using data from a large urban district, this study investigated whether racial inequality in access to eighth-grade algebra is a reproduction of differences in prior opportunities to learn (as evidenced by grades, test scores, and level of prior mathematics course) or whether patterns reflect an increase in inequality such that racial differences in access remain when controlling for academic background. We considered how this varies by the racial composition of the school; further, we examined differences in access between both Black and Hispanic students and their White peers as well as differences between Black and Hispanic students. The results point to patterns of reproduction of inequality in racially integrated schools, with some evidence of increasing inequality in predominantly Hispanic schools.

Keywords: Eighth-grade algebra; Opportunity to learn; Race; School racial composition

Educational practitioners, policymakers, and researchers have long recognized the influence that high school course-taking has on educational as well as labor force outcomes (Adelman, 1999; Gamoran, 1987; W. Tyson, Lee, Borman, & Hanson, 2007). In recent years, interest has also been paid to middle school, where eighth-grade algebra has been identified as the gatekeeper to favorable outcomes in high school and beyond. Specifically, eighth-grade algebra completion predicts not only subsequent advanced mathematics course-taking (e.g., completing calculus in high school) but is also linked to a greater likelihood of enrolling in a 4-year college or university as well as an increased likelihood of pursuing a Science, Technology, Engineering, or Mathematics (STEM) major (Chen & Weko, 2009; Filer & Chang, 2008; Gamoran & Hannigan, 2000; Ma & Wilkins, 2007; Paul, 2005; Schneider, Swanson, & Riegler-Crumb, 1998; X. Wang, 2013).

As recognition of the benefits of early algebra course-taking has grown among educational policymakers as well as parents, there has been a nationwide surge in the percentage of students taking eighth-grade algebra (Stein, Kaufman, Sherman, & Hillen, 2011); according to Loveless (2016), the percentage of students taking algebra or higher in eighth grade has increased from 27% in 2000 to 48% in 2013. Unfortunately, these enrollment trends do not translate across all populations of student groups; it is well documented that Black and Hispanic students are

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Incorporating a Digital Game Into the Formal Instruction of Algebra

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We compared students' learning gains in authentic seventh-grade classrooms ($N = 144$) in 4 different interventions that incorporated a computer game that aims to teach players to solve linear equations. Significantly higher learning gains were measured in the implementations that were specifically designed to mediate the attribution of algebraic meaning to objects, actions, and rules in the game by engaging students in analogical mapping between these constructs and their algebraic counterparts and an exploration of the boundaries of this isomorphism. These findings suggest that learning disciplinary content and skills from a digital game requires learners to attribute disciplinary meaning to objects, actions, and rules in the game. Moreover, this process does not necessarily occur spontaneously and benefits from instructional mediation.

Keywords: Algebra; Analogical transfer; Game-based learning; Mathematical meaning; Pedagogy

Although numerous digital educational games have been marketed as resources with promising instructional potential, these games are rarely used systematically in schools (Proctor & Marks, 2013). Despite the concerted research and development efforts devoted to studying game design (Mayer, 2011), very little is known about how best to integrate these games into the classroom, and not enough attention has been paid to the features involved in integrating games (or technology in general) into formal instruction (Bielaczyc, 2006). Moreover, teachers play a pivotal role in integrating games into formal instruction (Foster & Shah, 2015). They are far more than mere facilitators, and the view that educational technology is something that simply needs to be implemented is not productive (Hoyle, Noss, Vahey, & Roschelle, 2013). This point is supported by a meta-analysis of 39 studies that examined learning from serious games (Wouters, van Nimwegen, van Oostendorp, & van der Spek, 2013), which showed that the cognitive and motivational effects of game-based learning were higher when the game was complemented by additional instructional activities. In the current study, we hypothesized that learning disciplinary content and skills from a digital game requires learners to attribute disciplinary meaning (in our case, mathematical meaning) to the corresponding objects, actions, and rules in the game and that this process does not necessarily occur spontaneously but can be enhanced by instructional mediation that corresponds to the game's core mechanics. In the following subsections,